

1. Write the equations (if possible) and explain the following terms as clear as possible.
 - (a) The Lorentz gauge and the Coulomb gauge. (4%)
 - (b) Gauge transformations and gauge freedom. (4%)
 - (c) Phase velocity and group velocity (4%)
 - (d) The two postulates of the special relativity (4%)
 - (e) Invariant quantity and conserved quantity. (4%)
 - (f) Lienard-Wiechert potentials. (5%)
 - (g) Hidden momentum (5%)

2. (a) The transformations between two inertial systems S and $\bar{S}$ are $\bar{x} = \gamma(x - vt)$ and $\bar{t} = \gamma(t - vx/c^2)$. Show that when $\Delta t = 0$, $\Delta x = \Delta \bar{x}/\gamma$; but when $\Delta \bar{t} = 0$, $\Delta \bar{x} = \Delta x/\gamma$. Explain why the length relations depend on the simultaneity. (8%+2%)
 - (b) Show that $(\mathbf{E} \cdot \mathbf{B})$ is relativistically invariant. (10%)

$$\bar{E}_x = E_x, \quad \bar{E}_y = \gamma(E_y - vB_z), \quad \bar{E}_z = \gamma(E_z + vB_y)$$
 [Hint: $\bar{B}_x = B_x, \quad \bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z), \quad \bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$]

3. (a) $\mathbf{r} = \mathbf{r} - \mathbf{r}'$, $r = |\mathbf{r} - \mathbf{r}'| = c(t - t_r)$, and $\mathbf{v} = \mathbf{v}(t_r)$. Find ∇r and $(\mathbf{r} \cdot \nabla)\mathbf{v}$. (10%) (Note: Express your answer in terms of ∇t_r .)
 - (b) Show that the retarded potentials satisfy the Lorentz gauge condition $\nabla \cdot \mathbf{A} + \mu_0 \epsilon_0 \partial V / \partial t = 0$. (10%) [Hint: the retarded potentials $V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{r} d\tau'$ and $\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{r} d\tau'$.]

4. Suppose $V=0$ and $\mathbf{A} = A_0 \cos(kx - \omega t) \hat{\mathbf{z}}$, where A_0 , ω , and k are constants.
 - (a) Find $\mathbf{E}$ and $\mathbf{B}$. (10%)
 - (b) Use the gauge function $\lambda = xt$ to transform the potentials V' and $\mathbf{A}'$. (10%)
 - (d) Find the new $\mathbf{E}'$ and $\mathbf{B}'$. Comment on the gauge freedom. (8%+2%)
 [Hint: $\mathbf{A}' = \mathbf{A} + \nabla \lambda$ and $V' = V - \partial \lambda / \partial t$]

